Cost Minimization Slides

Econ 360

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Learning Outcomes

- Compare and contrast profit maximization and cost minimization.
- Identify whether profit maximization and/or cost minimization is appropriate for a given situation.
- Apply cost minimization graphically and algebraically to a producer's problem.

Where We Are

- We can solve a firm's profit maximization problem.
- BUT we know profit maximization only gives a single input, output bundle if the firm's production function exhibits decreasing returns to scale.
- Cost minimization gives us a way to figure out how much a firm should produce, and how much of each input to use, even if the firm's production function exhibit increasing or constant returns to scale.

Notation Reminder: Technology

- Firms turn **inputs** into **output**.
  - We use y's to denote outputs. I.e. $\{y_1, y_2, ..., y_n\}$.
  - These outputs have prices $\{p_1, p_2, ..., p_n\}$.
  - We use x's to denote inputs. I.e. $\{x_1, x_2, ..., x_m\}$.
  - These inputs have prices $\{w_1, w_2, ..., w_m\}$.
- In this class, we will mostly focus on one output y.
- We will also generally focus on two main inputs, **L**abor, or workers, and **K**apital, or machines/buildings/land/any non-labor input.
  - Econ decided that K should be for capital since we use C for costs generally.
  - We will use w for the price of Labor and r for the price of K/Capital.

Review of Profit Maximization

- We drew isoquants (bundles of inputs that gave use the same output).
- We also drew isoprofit lines (bundles of inputs that gave us the same profit).
- We figured out the highest isoprofit line (objective) subject to our production function (isoquants, constraint).

Cost Minimization Overview

- For cost minimization, we are still going to use our isoquants.
  - Isoquants will still serve as the constraint.
- We will introduce isocost curves (bundles of inputs with the same cost).

- These isocost curves will be our objective, where we seek to figure out the lowest cost way to stay on a certain isoquant line.

The Isocost Curve

- In a general case with n inputs $x_1, x_2, ..., x_n$ and input prices $w_1, w_2, ..., w_n$ the cost function is
$$\Sigma_{i=1}^{n} w_i \cdot x_i.$$

- In our example with two inputs L and K with prices w and r:

- Cost=$wL + rK$.

- An isocost line would therefore be something like $100 = wL + rK$.
  - This is the isocost line or isocost curve where cost is equal to 100.

The Isocost Curve-Slope

- Suppose we consider an isocost curve with cost=$\bar{c}$.
- We can rewrite the isocost curve using one input as the left hand side variable.

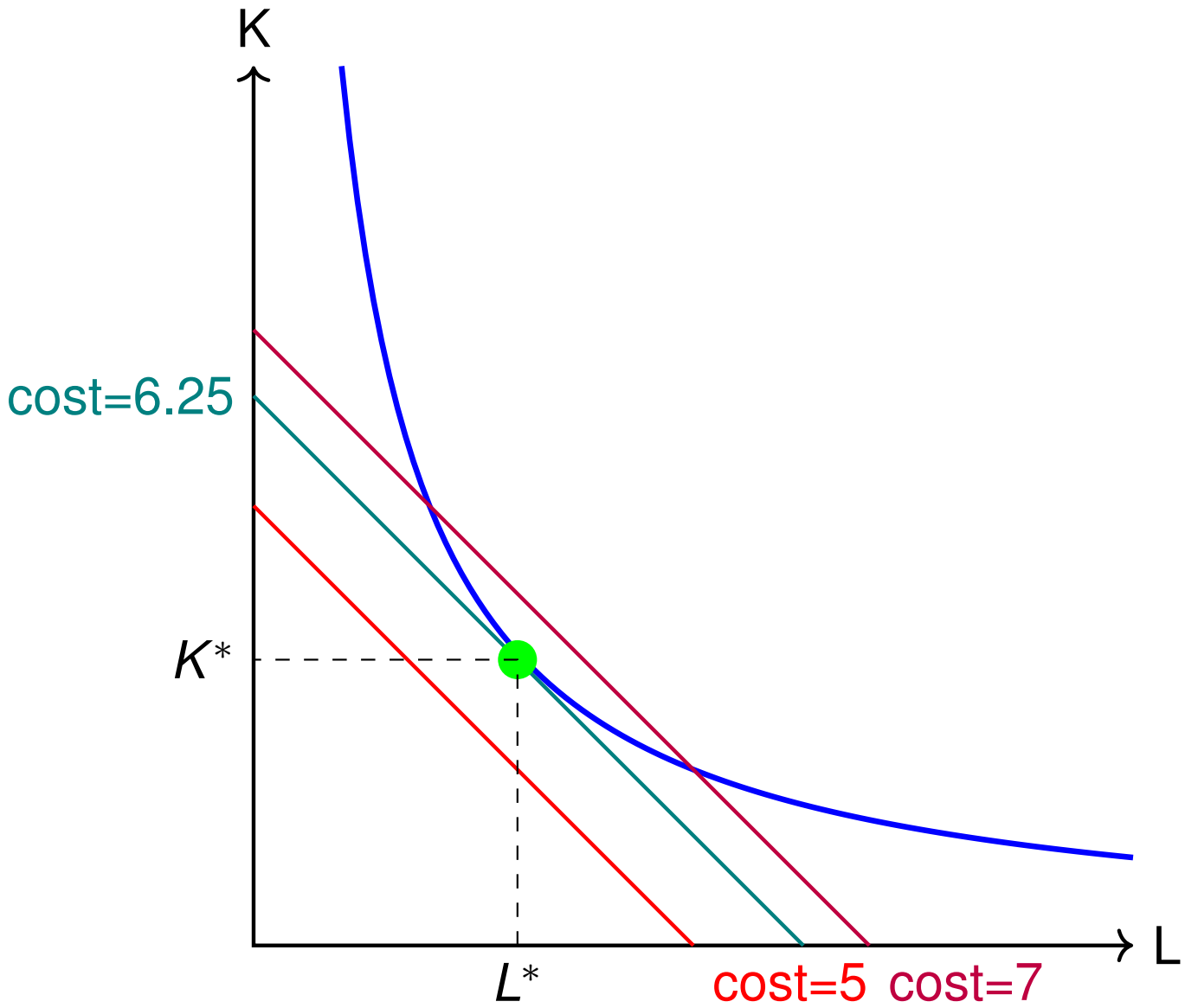
$$\begin{aligned} \bar{c} &= wL + rK \\ \bar{c} - wL &= rK \\ rK &= \bar{c} - wL \\ K &= \frac{\bar{c}}{r} - \frac{w}{r}L \\ K &= -\frac{w}{r}L - \frac{\bar{c}}{r} \end{aligned}$$

- This means that the slope of the isocost curve is simply the ratio of input prices!
- If you solved this equation for L, the slope would be $\frac{r}{w}$ which is still the ratio of input prices!

Cost Minimization–Graphed

- We will take a given isoquant (i.e. we will fix quantity at $\bar{q}$.
- We will then draw a bunch of different isocost curves.
- Our cost minimizing bundle of inputs (L^*, K^*) will be on the lowest possible isocost such that we are still on the isoquant curve we want!

Cost Minimization–Graphed



Cost Minimization: Marginal Benefit and Marginal Cost

- This shows us graphically our cost minimizing bundle is at the tangency of the isoquant curve and the isocost curve.
- We know the slope of the isoquant is simply the MRTS or TRS or Technical Rate of Substitution.
  - The TRS is just the ratio of the marginal products of the two inputs.
- We also know the slope of the isocost is just the ratio of the input bundles.
- So equating them gives

$$TRS = \frac{MU_K}{MU_L} = \frac{r}{w} = \text{Ratio of Prices.}$$

- Which is still just marginal benefit=marginal cost for both inputs!
- AND the same condition we had earlier for profit maximization.

Cost Minimization vs Profit Maximization

- Because we are fixing quantity, we do not need the production function to exhibit decreasing returns to scale.
- Therefore cost minimization will work even when profit maximization does not give us a well-defined maximizing bundle of inputs and an output level.
- Cost minimization also gives us a way to start thinking about cost functions.

From Cost Minimization to Cost Curves

- Eventually we want to say things about a firm's costs for any quantity.
- What we could do is change the isoquant we look at for cost minimization.
- For each quantity we simply find the optimal bundle of K and L.
- We find the cost of that bundle as $wL^* + rK^*$.
- This becomes a cost of a given output level q.
  - This is called the **Expansion Path**.
- So we can write a cost function $c(q)$!
  - We can do this for both the short and long run.